A letter from the creator of Arithmetickles:

Dear teacher:

When I was a child in elementary school, I remember being really frightened of math. Numbers were like a foreign language to me; they never seemed to make any sense. I couldn t relate addition and subtraction, multiplication and division to the real world. In fourth grade our new math teacher began the year by playing games with us.

Games? I couldn t believe that we were allowed to play games in school. And not only games; magic tricks as well. I didn t realize at the time that I was finally learning the power of math. It was too much fun.

The show Arithmetickles is based on audience participation, so every show is different. You can use the study guide that follows (a collection of great games and magic tricks all relating to math) in two (or even three) ways:

1) before the performance so that your students will be in the mood for a show that makes math fun, or
2) after the performance to demonstrate, once again, how much fun the world of math can be. or
3) both!

The games in the study guide are on three different levels of difficulty, as you will see. The How it Works part is to help teachers become great performers. The Why it Works part (for which I d like to thank our good friend, Leonard A. Smith, a health physicist from Carlisle, Massachusetts, who served as our mathematical consultant) is to satisfy the curiosity of the math lovers among us.

Sincerely,
Ben Bendor

P.S. We would love to hear from you and from your students as well. Please write or email us with any comments, anecdotes, questions or suggestions relating to the study guide that you may have.

THE DIVINE NINE

Math skills: Addition and multiplication
You will need: Pencil and paper
Difficulty level: 1

Introduction:

- ♦ Ask your students to do the multiplication in the next table.
- ♦ Now ask them to add the sums together and write the total under the "add" column. It's a great way to learn about patterns.

Multiply	add
1x9= 9	9
2x9= 18	1+8=9
3x9= 27	2+7=9
4x9=	
5x9=	
6x9=	
7x9=	
8x9=	
9x9=	

- ♦ Ask them if they think that it is also going to work for large numbers.

multiply	add	Multiply	add
29x9= 261	2+6+1= 9	34x9=	
30x9= 270	2+7= 9	35x9=	
31x9= 279	2+7+9=18/ 1+8= 9	36x9=	
32x9=		37x9=	
33x9=			

How (and why) it works:
The series 9x1, 9x2, 9x3, 9x4 can be written as a series of addition:
9, 9+9, 9+9+9, 9+9+9+9 i.e. the series is generated by adding 9 to the previous number. This is the same as adding 10 and subtracting 1.
This will result in no change, and since the first number in the series is 9 the other numbers in the series will be 9 as well.

THE GREAT EIGHT

Math skills: Addition and multiplication
You will need: Pencil and paper
Difficulty level: 1

Introduction:

- Ask your students to do the multiplication in the next table.
- Now ask them to add the sums together and write the total under the add column. It s a great way to learn about patterns.

multiply	add
1x8=	
2x8=	
3x8=	
4x8=	
5x8=	
6x8=	
7x8=	
8x8=	
9x8=	

- Ask them if they think that it is also going to work for large numbers.

Multiply	add	multiply	add
29x8=		34x8=	
30x8=		35x8=	
31x8=		36x8=	
32x8=		37x8=	
33x8=			

Why it works?

The series 8 x 1, 8 x 2, 8 x 3, 8 x 4, etc. can be written as a series of additions: 8, 8 + 8, 8 + 8 + 8, 8 + 8 + 8 + 8, i.e. the series is generated by adding 8 to the previous number. This is the same as adding nine and subtracting one. However, as we already know, adding nine makes no change to the sum of the integers. Therefore the net result of adding nine and subtracting one is to reduce the successive members of the series by 1. However, when the series reaches 1 the next number is not 0 but 9. For the number to be 1 the successive sums of the integers must be of the form 10, 100, 1000, etc. If the original number is reduced by 1 the sums of the integer will be 9, 99, 999, etc. which all successively sum to nine. Hence this series reduces by 1 until it reaches 1 and then goes to 9 and so on.

READING YOUR MIND!

Math skills: Addition
You will need: Four strips of cardboard
Difficulty level: 1

Introduction: Tell your students that you can guess the number they are going to choose in 2 seconds!

What to do:

- Prepare four columns of numbers headed A, B, C, and D as shown here.
- Present the columns to the class, and ask a volunteer to choose a number between 1 and 15. Have the volunteer write this number on the board (without you seeing it).
- Ask the volunteer to look at the columns and tell you each column where the number appears (A, B, C, or D)
- Ask your class to concentrate and then you announce the number they chose!

A	B	C	D
8	4	2	1
13	7	10	5
15	13	3	11
11	5	6	3
10	12	11	13
14	15	14	15
9	14	7	9
12	6	15	7

How it works:
All you have to do is add the sum of the top numbers in each column that the volunteer announced. Let's say that the number was 7. Your volunteer will see that 7 appears in columns B, C, and D. You then add up 4+2+1=7. It is going to work with every other number as well! Now, if you really want to impress everyone, memorize the top numbers (A=8; B=4; C=2; D=1) in the columns and then do it without looking at the strips!

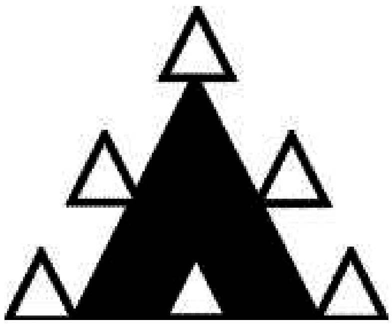
Why it works:
The first row of numbers, 8, 4, 2 and 1 were assigned to columns A, B, C and D respectively. Other numbers, 3, 5, 6, 7, 9, 10, 11, 12, 13, 14, and 15 were assigned to selected columns so that the sum of the numbers in the first row of the selected columns equal the assigned number. For example: 3 would be assigned to columns C and D because the number in the first row of the columns C and D are 1 and 2 which sum to 3.

Note that A, B, C, and D are really the 8, 4, 2 and unit places of the binary number system (all 2 number system). All the decimal numbers are placed in columns corresponding to their binary analog. For example the base 10 number 13 is written as 1101 in binary form which is 8 +4 +1 =13 and 13 is assigned to columns A, B, and D while C is left empty.

THE MAGICAL TRIANGLES!

Math skills: Addition
You will need: Paper and pencil
Difficulty level: 2

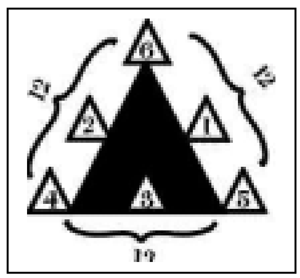
Introduction: There is only one way to fit the numbers in this triangle so that all the sides are equal.



What to do:

- ♦ Show your class this triangle and ask them to try to fit the numbers 1,2,3,4,5,6 into the small triangles so that the sum of each side of the triangle will be exactly **12**.
- ♦ Ask them to try again with a different set of numbers: 4,5,6,7,8,9. The sum will now be **21**.
- ♦ And what about 2,4,6,8,10,12 (sum of **24**)? Ask them to find a pattern and create their own magical triangle!

How it works:
Place the highest three numbers in the corner spots. Then place the lowest number between the two highest; the next lowest between the next two highest and the third lowest in the remaining place. The sum of the sides will always be the same!



Why it works:

The chosen numbers form a series A, A+1, A+2, A+3, A+4, A+5.
The three highest numbers, A+5, A+4, and A+3 are assigned to each corner of the triangle. The three lowest numbers are assigned to the sides of the triangles with the lowest number between the two highest; the next lowest between the next highest and the remaining numbers on the remaining side. The sum of the sides are then given by:

$$A+3+A+1+A+5=3A+9$$
$$A+5+A+A+4=3A+9$$
$$A+4+A+2+A=3=3A+9 \text{ (which are all equal).}$$

Note: This also works if you reverse the procedure and put the smallest number in each corner and place the largest numbers between the two smaller, etc. The numbers do not have to be consecutive, but simply have the same interval between them; hence, the series A, A+X, A+2X, A+3X, A+4X, A+5X will also work. Also the procedure will work if the highest 3 numbers are separated by any interval from the three lowest numbers; for example the series A, A+X, A+2X, B, B+X, B+2X will work.

DOMINO MAGIC

Math skills: Addition and multiplication
You will need: A box of dominos; a calculator
Difficulty level: 2

Introduction: Tell your students that it s possible to do math problems in your head faster than with a calculator.

What to do: Ask a student to choose one domino (without a blank space), show it to the class (without showing it to you) and ask them to do the next steps on their calculators:

- ♦ Multiply one of the numbers by 5
- ♦ Add 7
- ♦ Multiply the result by 2
- ♦ Add the other number on the domino
- ♦ Ask your volunteer to write the answer on the board
- ♦ You write the 2 numbers on the original domino on the board!

How it works: Each domino has two numbers A and B, which can each have any of the following values: 1,2,3,4,5 and 6.
Let s say that the chosen domino had the numbers 5 and 2, and the student chose the number 5.

- ♦ Multiply A by 5: 5A
- ♦ Add 7: 5A + 7
- ♦ Multiply by 2: (5A + 7) x2 = 10A + 14
- ♦ Add B: 10A + 14 + B
- ♦ Subtract 14: 10A + 14 + B- 14= 10A + B

Why it works:
Since A and B are both units, 10A will be a ten and B will be a unit. Therefore, A and B can be determined. *(Note that this procedure also works with blanks or when A and/or B are zero.)*

QUICK ADDITION WITH THE CALENDAR

Math skills: Addition and multiplication
You will need: A calendar and a calculator
Difficulty level: 2

Introduction: Tell your class that you can add and multiply faster than the best calculator in the world!

What to do:

- ♦ Ask your volunteer to choose any full week on the calendar (meaning a week with seven days; the month doesn t matter) without showing it to you or to the class.
- ♦ Explain that the class will have to add the sum of the seven days of the week on their calculators.
- ♦ On the word go , have the volunteer announce out loud the date of the first day of the week that he chose. The first one to get the answer should announce it out loud.
- ♦ Since you will be the first one every single time, write it on a pad that the students can t see. When the first correct answer is yelled out, show your students that you got it long before they did!

How it works:

- ♦ Let s say the volunteer chooses the week of December 5th through December 11th.
- ♦ Add the number 3 to the first day (5+3=8). Multiply the sum by 7 (7x8=56). That s your answer. (5+6+7+8+9+10+11=56)

Why it works:
If the first day of the week is A, then the days of the week are A, A+1, A+2, A+3, A+4, A+5 and A+6.
Add the seven days: A + A+1 + A+2 + A+3 + A+4 + A+5 + A+6
=7A +21
=Ax7 + 3x7
=(A+3) x7

CUT THE CLOCK

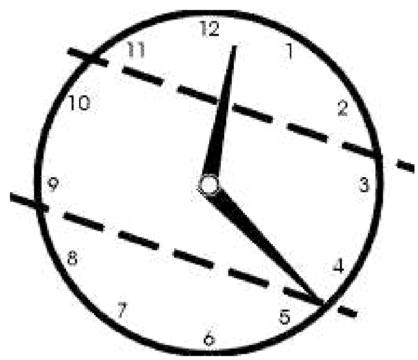
Math skills: Addition
You will need: Pen and paper
Difficulty level: 2

Introduction: We are going to "cut" the clock into 3 equal parts!

What to do:

- Draw a clock on the board.
- Ask your students to draw one on a piece of paper.
- Ask them to divide the clock with two straight lines (not crossing each other) so the sum of each part will be equal.

How it works:
When you combine all the numbers on the clock you will get 78.
Now you have to find 3 equal parts that will add up to 78:3= 26.



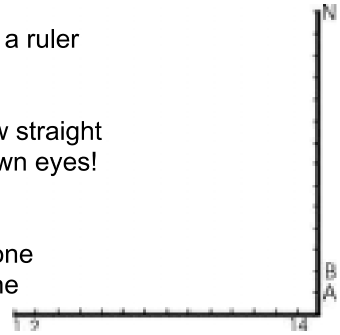
Why it works:
The numbers on the clock form the series 1,2,3,4,5 12. If you draw a line from between 12 and 1 to between 6 and 7, pairs of numbers opposite this line sum to 13, i.e. 12 + 1, 11 + 2, 10 + 3, etc. When you cut the clock in 3 sections there must be 12:3 = 4 numbers in each section, or two pairs in each section. Each pair adds to 13; 2 pairs add to 26.

DON T TRUST YOUR EYES!

Math skills: Measurements
You will need: Pen, paper and a ruler
Difficulty level: 2

Introduction: We are going to draw straight lines that will curve in front of our own eyes!

What to do:
Ask your students to draw 2 lines, one horizontal and one vertical. Each line should be 7.5 inches long.

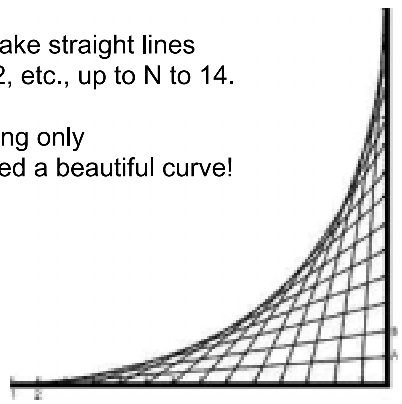


Now mark 14 equal points on each line (0.5 inch between each one of them, ignore the corner).

Label the horizontal marks 1, 2, 3 14 and the vertical marks A, B, C N as shown in the diagram.

With the ruler, ask them to make straight lines attaching points A to 1, B to 2, etc., up to N to 14.

They will then see that by using only straight lines they have created a beautiful curve!



How (and why) it works:

The "curved line" is, of course, an illusion. It is really made up of a sequence of short, straight lines. However, the brain perceives this as a curve. This is because the brain matches sensory images from the eye (and other sensory sources) against images stored in the memory. In this case, the sensory image closely resembles a curve and the brain interprets it that way. Even when you know that the curve is really a sequence of straight lines, it is hard to get the brain to perceive each separate straight section.

THREE-NO REMAINDER-GUARANTEE!

Math skills: Division, addition
You will need: Pen and paper
Difficulty level: 2

Introduction: Tell your students that you can predict if a number divided by 3 will have a remainder or not, without a calculator!

What to do: Ask a volunteer to write a phone number or a zip code on the blackboard. Before he/she finishes writing the number you are going to tell if the number will divide by 3 with or without a remainder. If the number has a remainder, you can offer him/her another number that will divide by the number 3 with no remainder.

How it works: If the volunteer wrote: 21657, adding the digits gives 2+1+6+5+7=21 which is divisible by 3 with no remainder as is 21657. If you can divide the results by three with no remainder, the answer is that the original number will also be divisible by three with no remainder! It is even more exciting - Try to mix the digits and you will see that all the combinations will divide by 3 with no remainder as well! (Try 12567; 56721;76215; 67152; etc.) If your volunteer chooses a number that you see will have a remainder when divided by 3, just offer to add a new integer that will cause the new number to divide exactly by 3. For example: If a volunteer wrote 65123, the sum of the integers is 6+5+1+2+3=17 which is one short of 18 that is divisible by 3. If you offer to add one to the number to obtain 651231, this new number is now divisible by 3 with no remainder. Note that all of the examples are five digits numbers but the process will work for any whole numbers.

Why it works: Think of all the numbers that are divisible by 3. They could be written as a series: 3, 6, 9, 12, 15, etc. Note that in each number of the series the digits add up to a multiple of 3. When these sums form multi-digit numbers, their successive sums eventually are 3, 6, or 9. The reason for this is:

- the first 3 numbers of the series 3, 6, 9 are divisible by 3
- the next number in the series, 12, is obtained by adding 3 to 9
- whenever 9 is added to a number, the sum of the digits do not change because adding 9 adds a ten and removes a unit.

Therefore the sum of the integers in the series will become 3, 6, 9, and 3 etc, always divisible by 3.